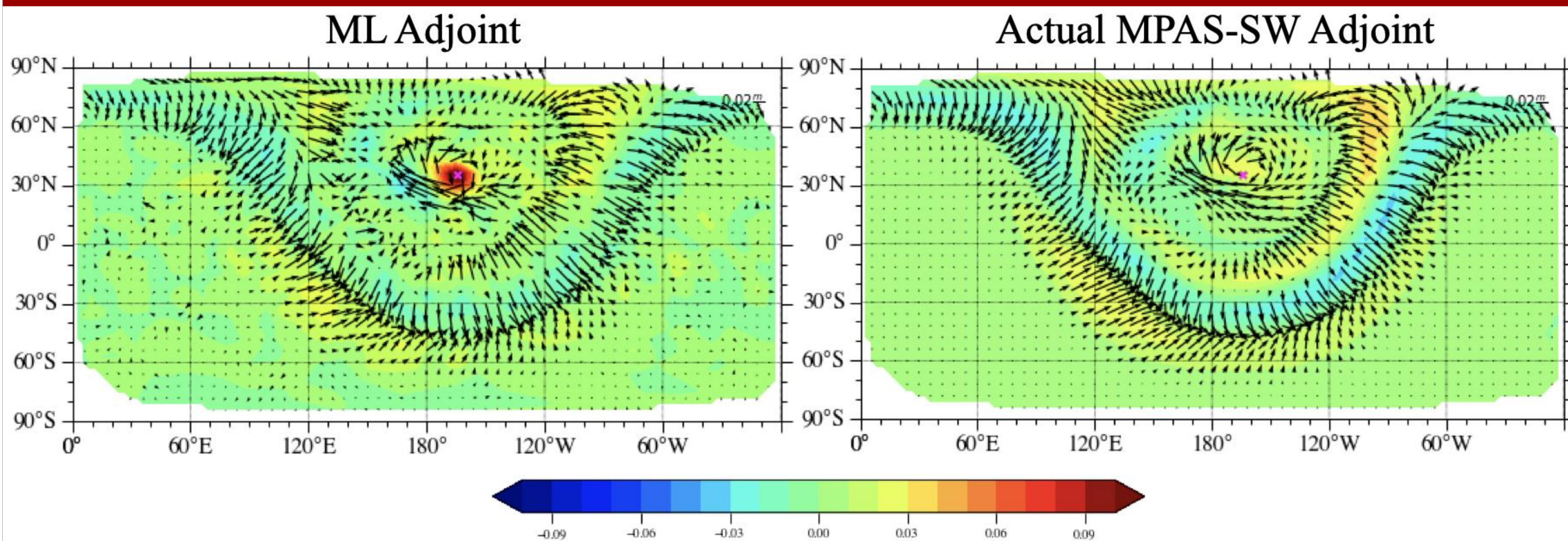


A Neural-Network Based MPAS—Shallow Water Model and Its 4D-Var Data Assimilation System

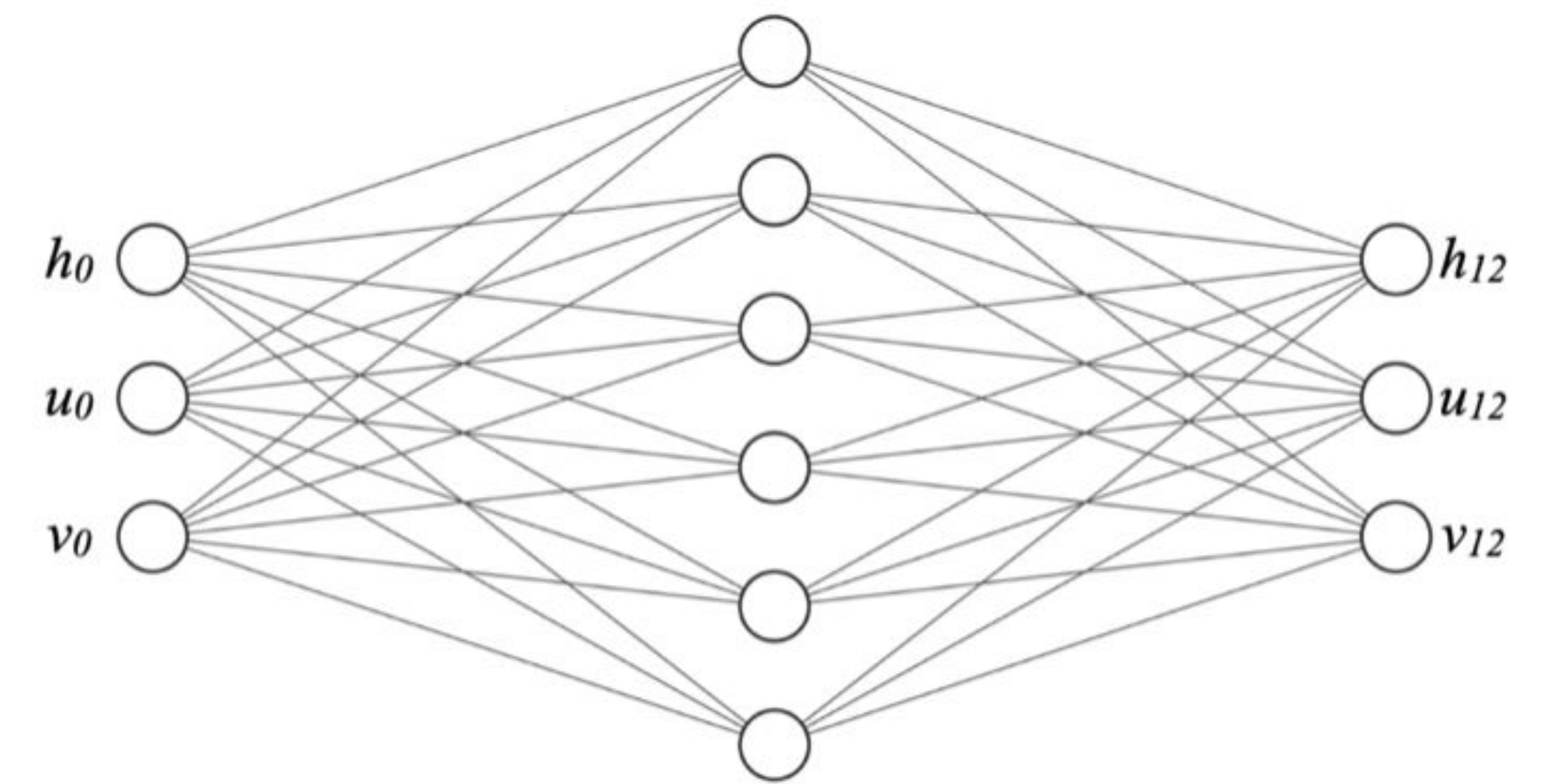
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Neural-network based MPAS-Shallow Water

$$\frac{\partial h}{\partial t} + \nabla(h\mathbf{u}) = 0,$$

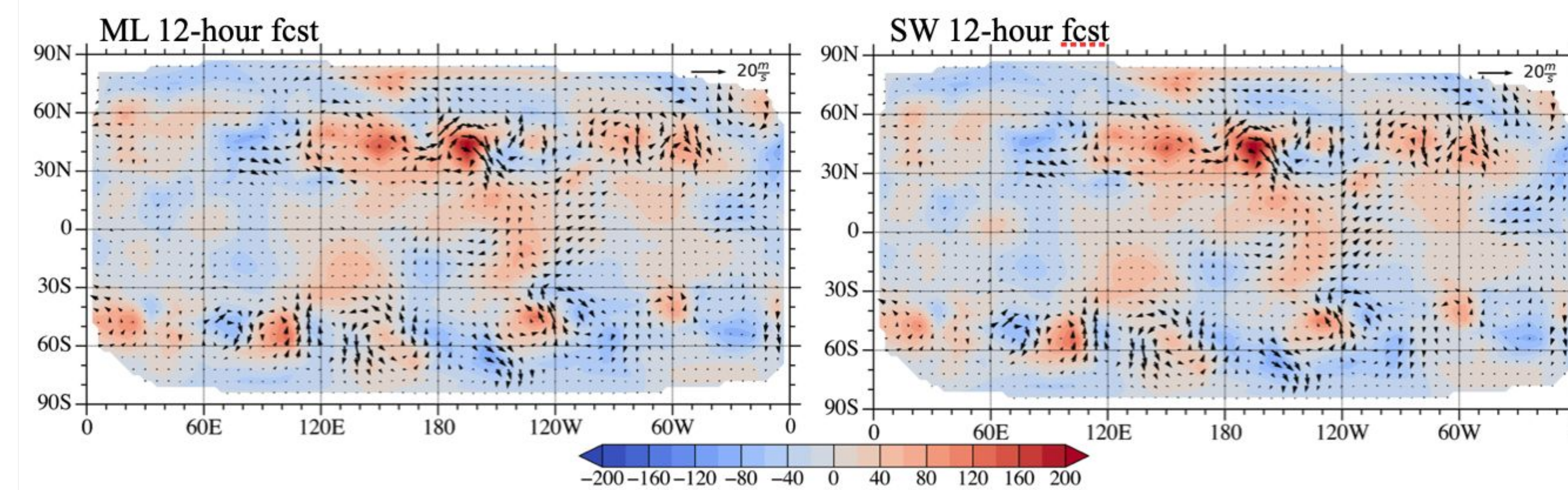
$$\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\nabla)\mathbf{u} + f\mathbf{k} \times \mathbf{u} = -g\nabla(h + b),$$



Input Layer $\in \mathbb{R}^3$

Hidden Layer $\in \mathbb{R}^6$

Output Layer $\in \mathbb{R}^3$



Tangent linear and adjoint in NN

Single layer NN

```
def pred_layer(x, w, b, acti):
    res = np.matmul(x, w) + b
    return acti(res)

def pred_layer_tl(dx, x, w, b, acti_tlm):
    res_tl = np.matmul(dx, w)
    res = np.matmul(x, w) + b

    return acti_tlm(res_tl, res)

def pred_layer_ad(x, dy, w, b, acti_adj):
    # --- forward calculation ---
    res = np.matmul(x, w) + b
    # --- end forward calculation ---

    res_ad = acti_adj(res, dy)
    return np.matmul(res_ad, w.T)
```

Multi-layer or deep NN

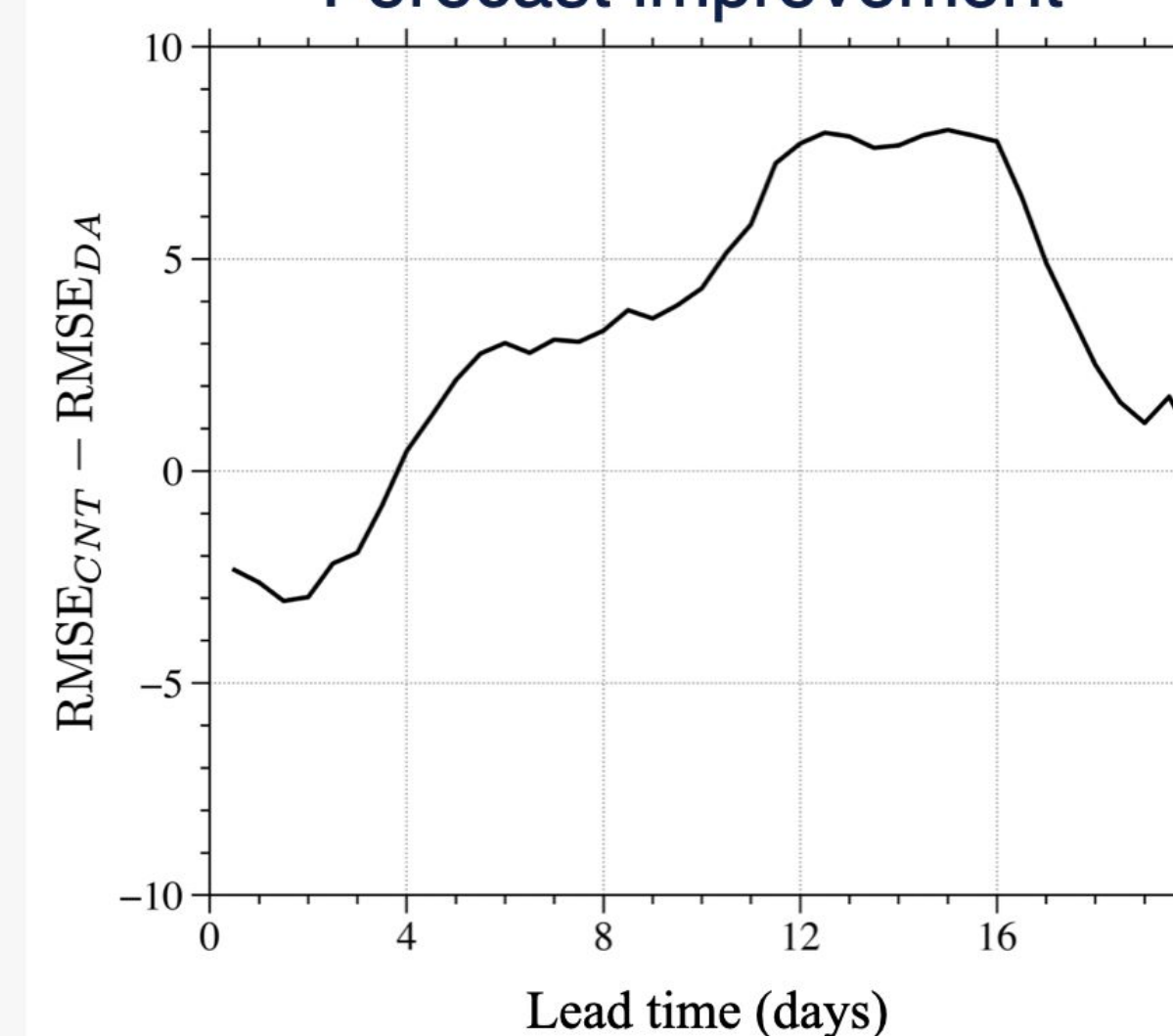
```
def pred(x, ws, bs, actis):
    ix = x.copy()
    for i in range(len(ws)):
        ix = pred_layer(ix, ws[i], bs[i], actis[i])
    return ix

def pred_tl(dx, x, ws, bs, acti_tlms):
    ix_tl, ix = dx.copy(), x.copy()
    for i in range(len(ws)):
        ix_tl, ix = pred_layer_tl(
            ix_tl, ix, ws[i], bs[i], acti_tlms[i])
    return ix_tl, ix

def pred_ad(x, dy, ws, bs, actis, acti_adj):
    # --- forward calculation ---
    ix = x.copy()
    x_fwd = []
    for i in range(len(ws)):
        x_fwd.append(ix.copy())
        ix = pred_layer(ix, ws[i], bs[i], actis[i])
    # --- end forward calculation ---

    iy = dy.copy()
    for i in range(len(ws)-1, -1, -1):
        # pred_layer_ad(x, dy, w, b):
        iy = pred_layer_ad(
            x_fwd[i], iy, ws[i], bs[i], acti_adj[i])
    return iy
```

Forecast improvement



Discussion and conclusions

- The potential next steps for this research are numerous. Similar techniques are readily applicable in substituting noise physics parameterizations or observation operators in a DA system
- The recent advances of ML applications in NWP are especially encouraging in this aspect
- These encouraging results demonstrate the feasibility of the tangent linear and adjoint components obtained from neural networks and the potential value of the proposed DA system



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