Linear approximation of the perturbation dynamics $\mathbf{q}(t) = \mathbf{A}(t{:}0)\ \mathbf{q}(0)$,

Growth rate of perturbations

Singular Vectors are the eigenvectors of , where **A** is the model operator.

Deutscher Wetterdienst
Wetter und Klima aus einer Hand

Singular Vector perturbations with Pangu-Weather

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In recent years machine learning models for weather forecasting (MLWP) have made huge progress and are now competitive to traditional numerical weather prediction (NWP). ML models can reproduce main atmospheric circulation patterns by learning some state space representation of the atmospheric dynamics. Hence, MLWPs should be aware of dynamic sensitivities in the atmosphere which may lead to fast error growth when appropriately perturbing the initial conditions. However, Selz and Craig [1] found that the Pangu-Weather model is not sensitive to butterflies, but synoptic-scale perturbations from NWP systems show the expected exponential growth.

Arnoldi Singular vectors (A-SV)

Singular vectors (SV) [2] constitute an orthonormal basis that approximates the perturbation space at a given initial state of the dynamical system. The forecast model is successively integrated within a prescribed optimization window and generates a new SV with each forecast. During this process the leading SV is expected to turn into the direction of the fastest local error growth.

For the calculation of Pangu-Weather SVs we use the Arnoldi method [3] (see Fig. 1), which is adjoint-free and is based on the full non-linear model. We initialize the A-SV iterations from Gaussian white noise perturbations of the wind components (U,V) and the temperature (T) on pressure levels and use the total energy norm to measure error growth.

Fig. 2 shows vertically integrated perturbation energies: the exponential increase of the total pert. energy with lead time for selected SVs (upper panel) and the spacial distribution of the kinetic pert. energy of the first right SV (lower panel). The line contours denote the 500hPa geopotential and demonstrate the flow dependency of the SV perturbations. Full details of the results will be given in [4].

The next step is setting up an MLWP ensemble system. In contrast to the GenCast concept [6] we initialize the ensemble from A-SVs consistent with analysis error to simulate flow dependent error growth from initial condition uncertainty (blue lines in Fig. 2) and add Gaussian noise during the forecast to account for random components of model error (black solid line in Fig. 2).

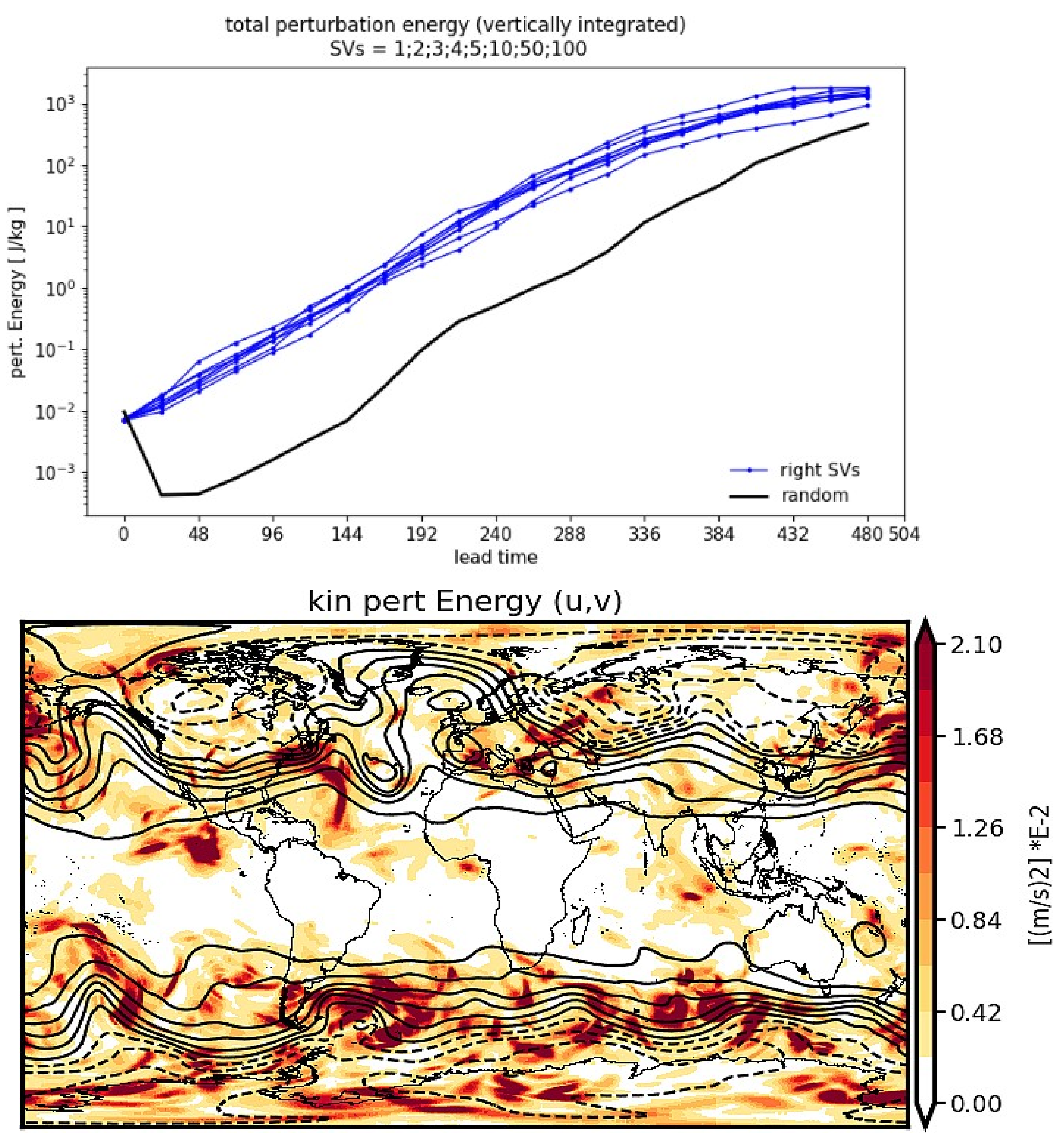

Figure 2: Example of total perturbation energy of selected global SVs (blue lines) and random perturbations (black line) in upper panel. The lower panel shows vertically integrated kinetic perturbation energy of the first global right A-SV. The line contours denote the 500hPa geopotential. The Pangu-Weather forecast initializes at 11 Jan 2024 00UTC.

Arnoldi SV algorithm

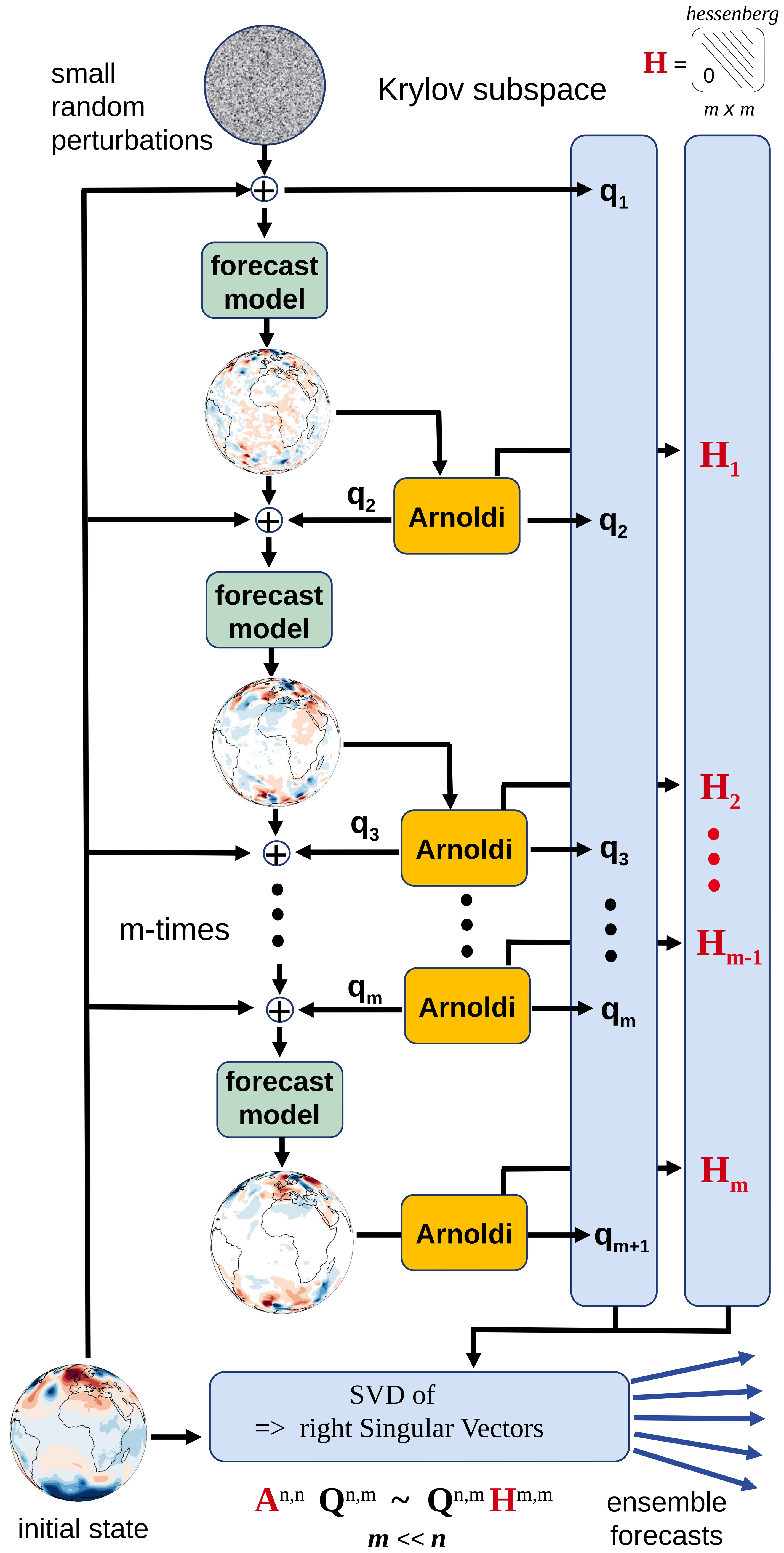

Figure 1: Illustration of the Arnoldi Singular Vector algorithm (A-SV), which is adopted to Fig 1 in [6] to facilitate comparison.

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WP2 of the **ECMWF** ML pilot project [5]

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